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Procedia Engineering00 (2013) 000–000

Procedia Engineering

www.elsevier.com/locate/procedia

12th International Conference on Computing and Control for the Water Industry, CCWI2013

Online Modelling of Water Distribution System using Data Assimilation

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Abstract

This paper applies Data Assimilation (DA) methods to a Water Distribution System Model to improve the real-time estimation of water demand, and hydraulic system states. A time series model is used to forecast water demands which are used to drive the hydraulic model to predict the future system state. Both water demands and water demand model parameters are corrected via DA methods to update the system state. The results indicate that DA methods improved offline hydraulic modelling predictions. Of the DA methods, the Ensemble Kalman Filter outperformed the Kalman Filter in term of updating demands and water demand model parameters.

Keywords: Data Assimilation, Ensemble Kalman Filter, Kalman Filter, Online Modelling, Water Distribution System

1. Introduction

The management of Water Distribution Systems (WDS) are devised to meet consumer demand with sustainable environmental and financial consequences. This means water demand forecasting is important component to help manage WDS [1]. However, to manage the WDS efficiently and effectively, short term and medium-term water demand forecasting is required to plan the regional water supply system [2]. These help water operation engineers to make better decisions concerning water supply balance [3]; planning and managing water demands during unplanned events [4] and setting optimal pumping schemes to reduce energy [1]. These strategy and contingency planning are done with the assistance of clean water hydraulic modelling software such as SynerGEE (Advantica), InfoWorks (Innovyze), WaterGEMS (Bentley) and EPAnet (US Environmental Protection Agency, (EPA)).

The hydraulic modelling software is mostly used off-line for specific objectives such as contingency planning, network optimisation and strategy planning [5]. To ensure there is a high confidence in off-line hydraulic models, off-line calibrations (based on short-term historical data) of the model are performed once every few years [5] i.e. United Utilities (UU) update their hydraulic models once in 5-10 years. The major drawback of off-line models is that both known and unknown parameters are updated by using short term sample of hydraulic data [6]. Therefore, the off-line calibrated model may not represent the current state of the WDS for operational purposes especially in emergency events [7].

The interest in developing online modelling of WDS has always been there [8] but the implementation of on-line modelling in a large scale is the major problem [6], [9]. The on-line hydraulic modelling is the combination of Data Assimilation (DA), hydraulic model and Supervisory Control and Data Acquisition (SCADA) to give a better representation of WDS. This makes operational parameters estimation more realistic through recursive and iterative processes [10].

However, Hatchett, et al. [10] highlighted that there are many developed data assimilation methodologies which are already applied to WDS modelling. For example, Shang, et al. [9] presented a Predictor-Corrector (PC) method to estimate water demand in real time. The PC method involved Autoregressive Integrated Moving average (ARIMA) [11] which is used to forecast the water demand (pattern values). The Extended Kalman Filter (EKF) is used to correct the predictions of water demand (pattern values) with the aid of observation of flow rate and head. Pries, et al. [6] used the PC framework and implemented M5 Model-Trees Algorithm [12] to predict the demand Multiplication factor (DMF) in real time. Then used Genetic Algorithm (GA) [13] with Huber function to correct the DMF based on the residual difference between model and predicted data (flow and pressure).

In this paper, offline hydraulic modelling is compared to online hydraulic modelling of a WDS to investigate whether DA methods can lead to improved predictions of water demand and system states (e.g. flow and pressure). A hydraulic model of a WDS is calibrated using system observations spanning one week, and use the observations of the following week to validate the WDS model calibration. The WDS model is run to predict the third week. During the third week the performance of the offline model is compared to predictions from the same model when applied using two data assimilation methodologies: The Kalman Filter (KF) and Ensemble Kalman Filter (EnKF), which are used to assimilate observations to update water demand estimates, and also the water demand forecasting model parameters.

2. Methodology

2.1. Offline Hydraulic Modelling

The modelling of WDS involves the use of both static asset information and dynamic parameters including demand distributions valve and pump operations. The WDS hydraulic model is calibrated manually by using one week (168hrs) of flow and pressure data. Only roughness values and demand profiles are altered during the calibration process. The demand profiles derived from the flow data of the following second week (169hrs – 336hrs) is used to model predictions (flow and pressure). These model predictions are compared to observed flow rates and pressure to check if the model is well-calibrated. When the model predictions match the observations, the first and second week demand profiles are used as 2 weeks demand pattern coefficient in the calibrated model. The calibrated model is then simulated for the 3 weeks and the third week model predictions (337 – 504hrs) are used as offline hydraulic modelling data.

2.2. Online Hydraulic Modelling

Online hydraulic modelling involves combination of Water Demand Forecasting Model (WDFM) and Data Assimilation (DA) methods to update the hydraulic state of the water distribution system. This process is regarded as predictor-corrector loop process. The steps of online modelling of WDS are as follow:

1. **State prediction:** this step is where the WDFM is run to forecast water demands for the next 15 minutes. These forecast water demands are then used to drive the hydraulic model from the known initial system state to next hydraulic states. The outputs from the hydraulic simulation are pipe flow rates, tank levels and pressures in the network.
2. **State correction:** DA methods (KF or EnKF) are used to update both forecast water demands and WDFM parameters. This method is driven by the difference in forecast hydraulics state (flow rates) and system observations at the current step.
3. Updated demands are then used to re-run the model to get the updated state at the current time step.
4. Repeat steps one to three at the next time step.

The initial online modelling starts at the beginning of third week (t = 336hrs).

3. Water Demand Forecasting Model

Water demand is one of the essential parameters to predict the WDS behaviour in real-time model. Therefore, there are Water Demand Forecasting Models (WDFM) available to forecast water demands such as ARIMA [11], M5 Model tress [12] and ANN [14]. Other models can be found in [1]. In this paper, a simple short-term WDFM is developed after experimentation of seasonal ARIMAs and regression analysis. The developed WDFM uses weighted rate of changes to forecast the water demand and is defined as:

$$d_t^f = M_t \, d_{t-1}^a \tag{1}$$

where d_t^f is the forecast demand at the current time step; M_t is the model operator and d_{t-1}^a is the updated demand at the previous time step (i.e. 15 minutes ago).

The model operator is the sum of the product of demand factor (rate of change) and its associated weight matrix:

$$M_t = \sum_{i=1}^{n_w} w_{i,t} \alpha_{i,t} \tag{2}$$

where n_w is the number of demand factors; $\alpha_{i,t}$ is the i-th demand factor and $w_{i,t}$ is the associated weight at time step, t.

The selected four demand factors are as follows:

$$\alpha_{1,t} = \frac{d_{t-1}}{d_{t-2}}; \qquad \alpha_{2,t} = \frac{d_{t-96}}{d_{t-97}}; \qquad \alpha_{3,t} = \frac{d_{t-672}}{d_{t-673}}; \qquad \alpha_{4,t} = \frac{d_{t-1344}}{d_{t-1345}} \tag{3}$$

where d_{t-1} is the demand from the previous 15mins, d_{t-2} is the demand from 30mins ago, d_{t-96} is the demand from one day (i.e. 24 hours) ago, day+15mins (t-97), week (t-672), week +15mins (t-673), 2 weeks (t-1344) and 2 weeks +15mins (t-1345).

The developed WDFM is a simple multivariate time series model which offers a mechanism of studying the impact rates of changes and associated WDFM parameters on demand estimation.

4. Data Assimilation

4.1. Kalman Filter with WDFM parameters updating

The Kalman Filter (KF) [15] is a recursive estimator that updates the forecast demands and WDFM parameters through the combination of forecast hydraulic states and system observations. The state estimation of the WDS system is expressed as:

$$\hat{y}_t = h(x_t^f) \tag{4}$$

where x_t^f are the forecast demands; $\hat{y}_t$ is the system observation(flow rates) and $h(.)$ is the nonlinear function relating observation to demands.

The KF is expressed in two steps, the analysis step where observations are assimilated into the filter, and the forecast step, where information about the system is used. The adapted procedure for updating forecast demands and WDFM parameters are as follows [16]:

$$\begin{cases} x_t^a = x_t^f + K_t^{xy}(y_t - \hat{y}_t) \\ w_t = w_{t-1} + K_t^{wx}(x_t^a - x_t^f) \end{cases} \tag{5}$$

$$\begin{cases} K_t^{xy} = P_t^{xy}\left(P_t^y + R_t^y\right)^{-1} \\ K_t^{wx} = P_t^{wx}\left(P_t^x + R_t^x\right)^{-1} \end{cases} \tag{6}$$

and the forecast step:

$$\begin{cases} x_{t+1}^f = M_t\, x_t^a \\ w_{t+1} = w_t \end{cases} \tag{7}$$

where x_t^a is the updated demands; y_t is the system observation; w_{t+1} and w_t are updated WDFM parameters at the time step respectively; K_t^{xy} and K_t^{wx} are the Kalman gain for updating demands and WDFM parameters respectively; P_t^y and P_t^x are the forecast error covariance matrix of forecast hydraulic state, $\hat{y}_t$ and demands, x_t^f respectively; P_t^{xy} is the cross covariance of forecast demands and system observations; P_t^{wx} is the cross covariance of WDFM parameters and forecast demands; R_t^y and R_t^x are the covariance of the system observations and updated demands respectively;

The difference $(y_t - \hat{y}_t)$ in equation 5 is called the Kalman innovation which reflects the discrepancy between the forecast hydraulic states and the system observations. The Kalman gains in equation 6 are the weight factor that uses a combination of observation and forecast error covariance. The problems with KF are: 1) It is very difficult to quantify the error covariance; 2) Kalman gain can give too much weight to forecast demand which can cause the divergence of the filtering process; 3) the correction process is restricted to residual error between the forecast and observed hydraulic state. Hence, KF in this paper uses an online single pass covariance [17] to update both the demand forecast error and WDFM parameter covariance:

$$\begin{cases} P_t^y = \left(P_{t-1}^y + (y_t - \hat{y}_t)^2\right)/2 \\ P_t^x = \left(P_{t-1}^x + (x_t^a - x_t^f)^2\right)/2 \end{cases} \tag{8}$$

where P_{t-1}^y and P_{t-1}^x are the forecast error covariance matrix of hydraulic states, $\hat{y}_t$ and forecast demand, x_t^f at previous time step, t-1 respectively;

This online algorithm for calculating the covariance is less prone to loss of precision caused by cancellation and also it considers previous updated covariance without storing all the historical covariance. However, the error covariance of the observations and updated demands are drawn from normal distribution with zero-mean and standard deviation of 1% of the observations. The cross covariance of hydraulic state and system prediction and the cross covariance of WDFM parameters and forecast hydraulic state [18] are expressed as:

$$\begin{cases} P_t^{xy} = H_t P_t^y \\ P_t^{wx} = P_t^w C_t^T \end{cases} \tag{9}$$

where H_t is the observation operator which maps the true state space into the observed space; C_t is the Jacobian of the WDFM model with respect to the model parameters and P_t^w is the variance of the WDFM parameters.

4.2. Ensemble Kalman Filter

Ensemble Kalman Filter (EnKF) [19] is a suboptimal estimator to update the ensemble of forecast demands and WDFM parameters separately without the need of covariance matrices. The analysis step of EnKF is:

$$\boldsymbol{X}_t^a = \boldsymbol{X}_t^f + \boldsymbol{K}_t^{xy}(\boldsymbol{Y}_t - \hat{\boldsymbol{Y}}_t) \tag{10}$$

and the general procedure of calculating Kalman gain is:

$$\boldsymbol{K}_t = \boldsymbol{C}_P(\boldsymbol{C}_P + \boldsymbol{C}_R)^{-1} \tag{11}$$

$$\begin{cases} \boldsymbol{C}_P = (N-1)^{\boldsymbol{-1}} \boldsymbol{E}_x \boldsymbol{E}_x^T \\ \boldsymbol{C}_R = (N-1)^{\boldsymbol{-1}} \boldsymbol{E}_y \boldsymbol{E}_y^T \end{cases} \tag{12}$$

$$\begin{cases} \boldsymbol{E}_x = \hat{\boldsymbol{Y}}_t - \mu_t^{\hat{y}} \\ \boldsymbol{E}_y = \boldsymbol{Y}_t - \mu_t^y \end{cases} \tag{13}$$

where N is the ensemble number; $\boldsymbol{w_t}$ and $\boldsymbol{w_{t-1}}$ are the ensemble matrix of updated and forecast WDFM parameters; $\boldsymbol{X}_t^a$ and $\boldsymbol{X}_t^f$ are the ensemble matrix of updated and forecast demands; $\boldsymbol{Y}_t$ and $\hat{\boldsymbol{Y}}_t$ are the system observations and predictions matrix; $\boldsymbol{K}_t^{xy}$ and $\boldsymbol{K}_t^{xw}$ are the Kalman gain for updating forecast demands and WDFM parameters; $\boldsymbol{C_P}$ is the cross covariance of ensemble states and predictions, $\boldsymbol{C_R}$ is the system observation error covariance; T is the transpose of the designated matrix; $\boldsymbol{E_x}$ and $\boldsymbol{E_y}$ are the forecast and observation errors; $\mu_t^{\hat{y}}$ and μ_t^y are the ensemble mean of forecast hydraulic states and system observation respectively.

In the application of the EnKF the assimilated observation is perturbed separately for each ensemble member. In the implementation presented here the perturbation is drawn from a truncated normal distribution with mean equal to the observation at each time step, and a variance equal to 0.25% of the observed values and limited to the range of 2% of the observed values. The ensemble of forecast hydraulics states are generated by perturbing the selected WDFM parameters (associated weights) with noise drawn from a truncated normal distribution at the initial step. The main principle of EnKF is to approximate the forecast and observation error covariance from these ensemble statistics in equation 12 and 13. The ensemble of WDFM parameters are updated by using the following KF steps:

$$\boldsymbol{w_t} = \boldsymbol{w_{t-1}} + \boldsymbol{K_t^{wx}}(\boldsymbol{\bar{x}_t^a} - \boldsymbol{\bar{x}_t^f}) \tag{14}$$

$$\boldsymbol{K_t^{wx}} = \boldsymbol{P_t^{wx}}(\boldsymbol{P_t^x} + \boldsymbol{R_t^x}) \tag{15}$$

$$\boldsymbol{P_t^{wx}} = \boldsymbol{P_t^w}\boldsymbol{C_t^T} \tag{16}$$

$$\boldsymbol{P_t^x} = \left(\boldsymbol{P_{t-1}^x} + (\boldsymbol{\bar{x}_t^a} - \boldsymbol{\bar{x}_t^f})^2\right)/2 \tag{17}$$

where $\boldsymbol{\bar{x}_t^a}$ and $\boldsymbol{\bar{x}_t^f}$ are ensemble mean of both updated and forecast demands; $\boldsymbol{\bar{w}_t}$ is the ensemble mean of WDFM parameters.

In equation 15, the covariance of the updated demand is drawn from normal distribution with zero-mean and standard deviation is 1% of the ensemble mean of updated hydraulic state. The WDFM parameter covariance is also drawn from normal distribution with zero-mean and standard deviation equal to 2% of the ensemble mean of WDFM parameters.

5. Case Study

The DA methods were tested on a real network which is renamed asWSZ01. WSZ01 has provides water service to approximately 16,000 customers. The WSZ01 model consists of 1 tank, 3 PRVs and 8 DMAs. All DMAs have one inlet and outlet flow meter except DMA03 which has two inlet flow meters. DMA01 has large percentage of industrial users and also covers a large retail park and a local airport. Figure 3 depicts the WSZ01 network configuration with sensor locations.

Table 1: The percentage of demand consumption in each DMA

Type of User	**DMA01**	**DMA02**	**DMA04**	**DMA06**	**DMA07**
Unmetered Domestic	58%	93%	95%	90%	92%
Metered Domestic	3%	0%	1%	5%	0%
10hrs Users	18%	5%	4%	5%	3%
24hrs Users	21%	2%	0%	0%	5%

In this case study, 3 DMAs in the network are not included because they are pressure managed. All the flow meters are located at the inlet of each DMA with 1 pressure sensor located at highest point in DMA01, DMA06. The model is calibrated based on 1 week observed data (flow rates and pressures) between 11th Feb and 17th Feb 2013. The observations between 18th and 24th Feb are used to validate the model calibration. Each DMA was grouped into 4 demand groups (table 1) to reduce the number of unknown parameters. The unmetered domestic users demand pattern coefficient and roughness values were modified to ensure that both predicted flow and pressure match the observed flow and pressure. Both offline and online modelling was then run for the remaining 7days (25th Feb – 3rd Mar 2013) with a 15 minute time step.

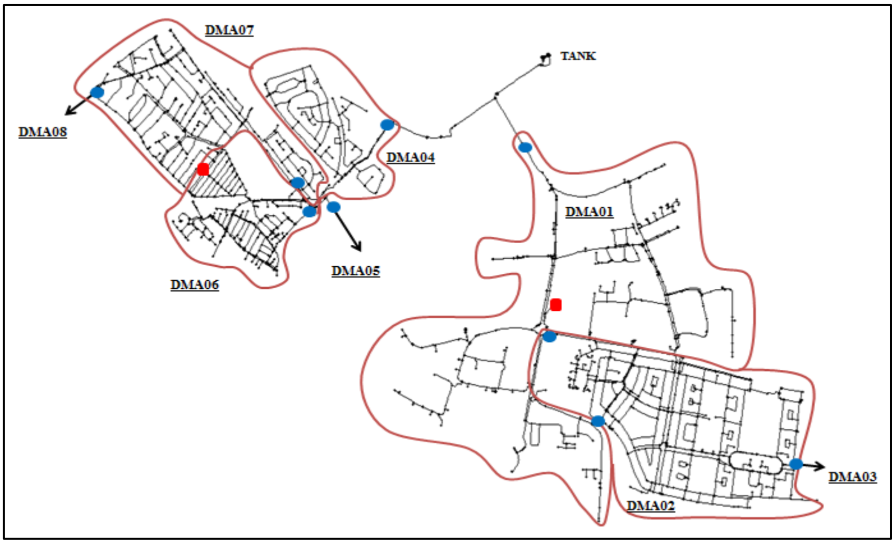

Figure 1:WSZ01 model with flow meter (blue dot) and pressure sensor (red square) locations

6. Results and Discussion

The number of assumptions is made in this paper: 1) pipe roughness values and other hydraulic model parameters are assumed to be known and remain constant during the online modelling; 2) the WSZ01 model have no leakage; 3) Only unmetered domestic users pattern coefficients are updated during online modelling while the other three demand group do not change during online modelling.

The value for WDFM parameters w_1, w_2, w_3 and w_4 are 0.2, 0.3, 0.3 and 0.2 respectively. These values are derived by using the mean absolute errors between the rate of change (demand factor) and its four selected rates of changes between 28th Jan 2013 to 24th Feb 2013. When EnKF is applied, the WDFM parameters ensemble was perturbed by adding noise drawn from a truncated normal distribution with mean equal to the WDFM parameters at each time step, and a variance equal to 2% of the values and range limited 20% of the values. The WDFM forecast water demands in each DMA are then disaggregated to compute the individual DMA unmetered domestic users' profile. The DA methods correct the water demands to update the hydraulic states in WSZ01 (figure 2). In the application of EnKF, an ensemble size of 10 members was used as predictive performance showed no sign of improvement with more members. The DA methods performed with the aid of EPANET and Microsoft Visual C++ on HP laptop (2.30GHz, 6.0GB of RAM). The execution time for each DA Method is displayed in table 2.

Table 2: Comparison of execution time for each data assimilation scheme

Hydraulic Modelling	Execution Time for a single time step	Execution Time for a week
Offline	Less than millisecond	0h 0m 4s
Online - KF	1.24s	0h 14m 01s
Online - EnKF	11.43s	1h 52m 48s

The results show that online hydraulic state prediction models performs better than offline hydraulic modelling accordingly to the table 3, 4 and Figure 3. This is because the DA methods update the forecast demands which are used to re-run the WSZ01 model to get the current hydraulic states of the system. These current hydraulic states of the system are then used as initial condition for the next time step. Both KF and EnKF have low Mean Absolute Error (MAE) values compared to offline values. Among DA methods, EnKF generally performed better the KF. The coefficient of determination values of DMA01 for online modelling is higher than the offline modelling because the DMA01 has higher percentage of industrial users which cannot be represented by the offline model.

Table 3: Comparison of the hydraulic modelling performances for each DMA demand prediction
MAE = Mean Absolute Error; R^2 = Coefficient of Determination. These statistics measure the distance between the observed and predicted demands in individual DMA.

Statistics	Method	DMA01	DMA02	DMA04	DMA06	DMA07
MAE(lps)	Offline	16.402	1.540	4.086	1.351	0.864
	KF	7.878	1.365	1.450	1.050	0.743
	EnKF	7.545	1.313	1.386	0.986	0.720
R^2	Offline	0.017	0.949	0.409	0.839	0.914
	KF	0.642	0.962	0.775	0.904	0.947
	EnKF	0.663	0.975	0.776	0.914	0.952

Table 4: Comparison of the pressure prediction statistics

Statistics	Method	DMA01	DMA06
MAE(m)	Offline	3.303	0.510
	KF	1.984	0.353
	EnKF	1.617	0.334

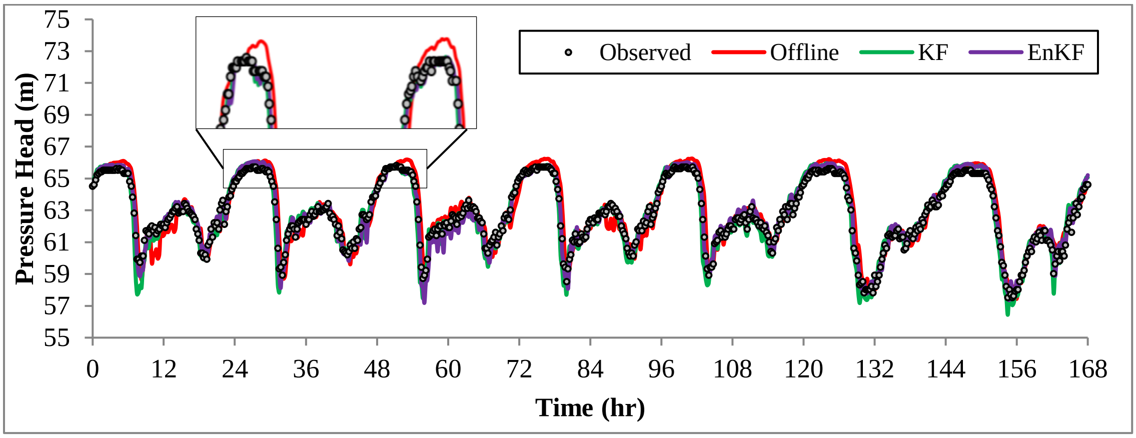

Figure 2: Comparison between observed and predicted pressure at node A0020A71 (DMA06) every 15mins

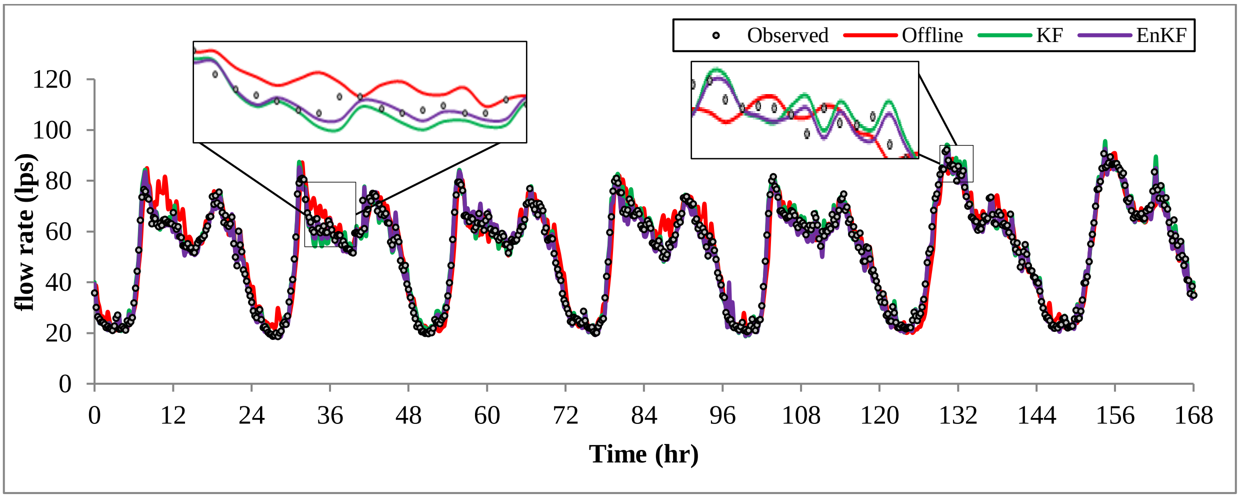

Figure 3: Comparison between observed and predicted flow rate at link X32230F7 (DMA04 flow meter) every 15mins ahead

Figure 4 shows that online modelling makes better predictions of flow rate in 15mins ahead compared to offline modelling. Ensemble mean of forecast demands tend to give better prediction compare to KF. There are various patterns of the updated parameter evolution between KF and EnKF and examples are displayed in figure 5 and 6. The noticeable pattern updated WDFM parameters between KF and EnKF is KF tend to have more irregular line compare to EnKF. Since the demand factors (rate of changes) in equation 3 change every 15mins, EnKF shows that WDFM parameters need to change steadily to give a good forecast of water demands in the next 15mins. The impact of the irregular pattern of WDFM parameters from KF affects the forecast demand in figure 4.

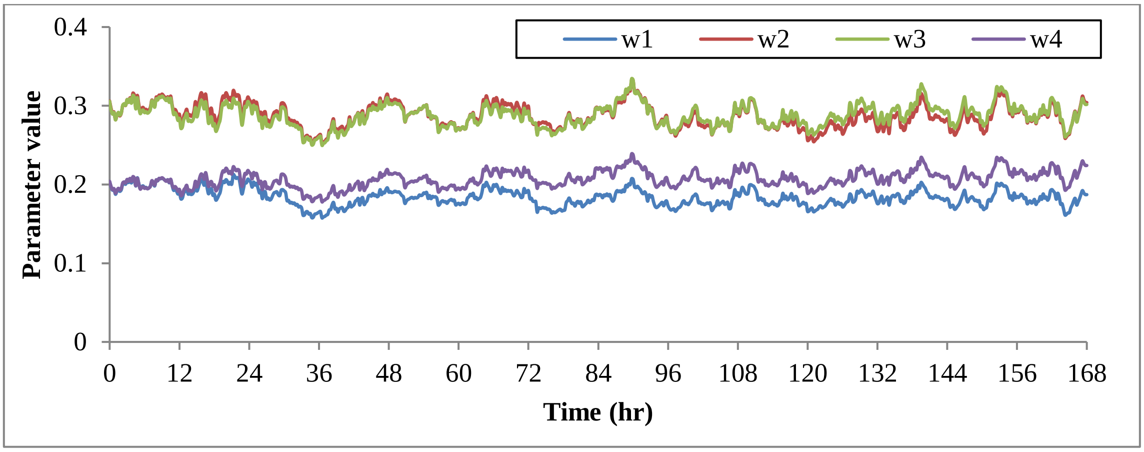

Figure 4: Evolution of updated WDFM parameter values in DMA05 (KF)

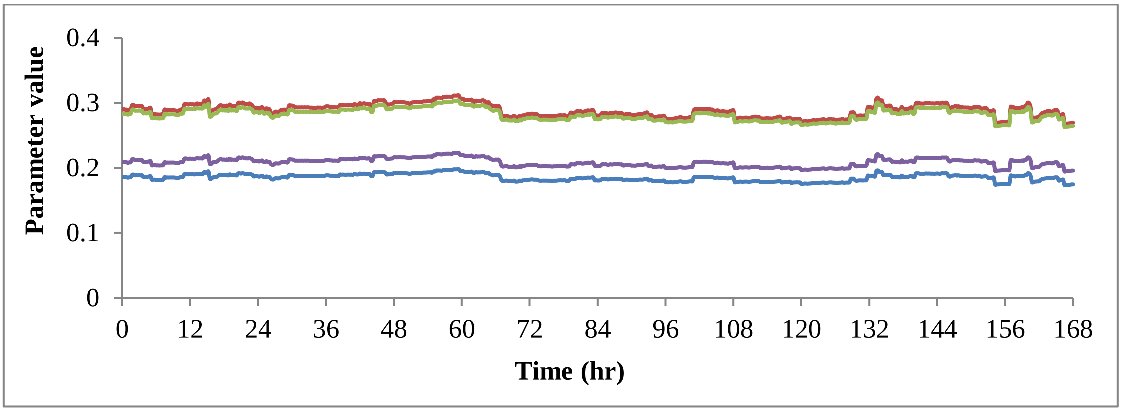

Figure 5: Evolution of ensemble mean of updated WDFM parameter values in DMA05 (EnKF)

7. Conclusion

On-line hydraulic modelling of a water distribution system is capable of making prediction that can reflect the WDS state more accurately than when using an offline model. This is because the DA methods used in an online model updates the system states which that minimises the bias in the initial conditions which, in turn, are used to simulate the system state in the next observation time step. Whilst the online model computational times are larger than the corresponding offline model run times. They are feasible for real time application.

The results obtained demonstrate that the EnKF performs well compared to the KF method in term of updating WDFM parameters. However, it takes KF 70% less of EnKF time to run online modelling of WSZ01 for a week. It is still feasible apply EnKF for online hydraulic modelling given the time step in real-time is 15 minutes.

Further research will include an investigation how both pressure and flow data could be used to update WDFM parameters and other 3 demand pattern coefficients (metered domestic user, 10hrs and 24hrs user).

Acknowledgements

The authors are grateful to United Utilities (UU), Mr D. Clucas, Mr T. Allen, Mr N. Croxton and UU hydraulic modelling team for providing the case study data and supporting financially the STREAM EngD project.

References

[1] Herrera, . M., Torgo, L., Izquierdo, J. & Perez-Garcia, R., 2010. Predicitive Models for Forecasting Hourly Urban WaterDemand. *Journal of Hydrology,* pp. 141-150.
[2] Zhou, S. L., McMahon, T. A., Walton, A. & Lewis, J., 2002. Forecssting Operational Demand for an Urban Water Supply Zone. *Hydrology,* Volume 259, pp. 189-202.
[3] Bougadis, J., Adamowski, K. & Diduch, R., 2005. Short-term Municipal Water Demand Forecasting. *Hydrological Processes,* Volume 19, p. 137=148.
[4] Jain, A. & Ormsbee, L. E., 2002. Short-term Water Demand Forecasting Modelling Techniques-Coventional versus Artificial intelligence. *American Water Works Association,* Volume 94, pp. 64-72.
[5] Machell, J., Mounce, S. R. & Boxall, J. B., 2010. Online Modelling of Water Distribution Systems. *Drinking Water, Engineering and Science.*
[6] Shang, F. et al., 2006. *Real Time Water Demand Estimation in Water Distribution System.* Cincinnati, Ohio, USA.
[7] Preis, A., Whittle, A. & Ostfeld, A., 2010. On-line Hydraulic State Prediction for Water Distribution Systems.
[8] Preis, A., Whittle, A., Ostfeld, A. & Perelman, L., 2011. Efficient Hydraulic State Estimation Technique Using Reduced Models of Urban Water Networks. *Water Resources Planning and Management.*
[9] Hutton, C. J., Kapelan, Z., Vamvakeridou-Lyroudia, L. & Savic, D. A., 2012. Dealing with Uncertainty in Water Distribution Systems' Models: A Framework for Real-Time Modelling and Data Assimiliation.
[10] Hatchett, S. et al., 2009. Real-Time Hydraulic Modeling: Open source software and results from a long-term field study.
[11] Box, G. P. & Jenkins, G. M., 1976. *Times Series Analysis: Forecasting and Control.* s.l.:Holden-Day.
[12] Quinlan, J. R., 1992. *Learning with Continous Classes.* Singapore, Proceedings 5th Asutrialian Joint Conference on Artificial Intelligence. World Scentific.
[13] Holland, J. H., 1975. *Adaptation in Natural and Artifical Systems,* s.l.: University of Michigan Press.
[14] Mounce, S., 2005. *A neural network approach to burst detection,* Ph.D. Thesis. Sheffield University: U.K.
[15] Kalman, R. E., 1960. A New Approach to Linear Filterig and Prediction Problems. *Transactions of American Socitey of Mechanical Engineers,* Issue Series D, 82, pp. 35-45.
[16] Moraradkhani, H. S., Sorooshian, S., Gupta, V. H. & Houser, P., 2005. Dual State-Parameter Estimation of Hydrologic Model Using Ensemble Kalman Filter. *Water Resources,* 28(2), pp. 135-147.
[17] Knuth, D. E., 1998. *The Art of Computer Programming: Seminumerical Algorithms,.* Third Edition ed. Boston: Addison-Wesley.
[18] Smith, P. J., 2010. *Joint state and parameter estimation using data assimilation with application to morphodynamic modelling,* Ph.D. Thesis. Reading University: U.K.
[19] Evensen, G., 1994. Sequential data assimilation with a nonlinear quasi-geostrophic model using Monte Carlo methods to forecast error statistics. *Geophysic Research,* 49(C5), pp. 10143 - 10162.